EXERCISE – V

JEE PROBLEMS

1. For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals 9/2 ? **[JEE 99,3]**

(A) – 4 (B) – 2 (C) 2 (D) 4

2. Find the area of the region lying inside $x^2 + (y - 1)^2 = 1$ and outside $c^2x^2 + y^2 = c^2$ where $c = \sqrt{2} - 1$. **[JEE 99,6]**

3. Find the area enclosed by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at (2, 3) and the x-axis. **[JEE 99,3]**

4. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is **[JEE 2002, (Scr.)]**

(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4

5. Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and $y = 2$, which lies to the right of the line $x = 1$ **[JEE 2002]**

6. If the area bounded by $y = ax^2$ and $x = ay^2$, $a > 0$, is 1, then a equals **[JEE 2004, (Scr.)]**

(A) 1 (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{3}$ (D) $-\frac{1}{\sqrt{3}}$

7. (a) The area bounded by the parabolas $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line $y = 1/4$ is **[JEE 2005, (Scr.)]**

(A) 4 sq. units (B) 1/6 sq. units
(C) 4/3 sq. units (D) 1/3 sq. units

(b) Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$. **[JEE 2005,(Mains),4+6]**

(c) If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$, $f(x)$ is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of $y = f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by $f(x)$ and chord AB.

8. Match the following **[JEE 2006,6]**

(i) $\int_0^{\pi/2} (\sin x)^{\cos x} (\cos x \cot x - \ln (\sin x)^{\sin x})\, dx$ (A) 1

(ii) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ (B) 0

(iii) Cosine of the angle of intersection of curves $y = 3^{x-1} \ln x$ and $y = x^x - 1$ is (C) 6ln2

(D) 4/3

9. (a) The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \frac{\pi}{4}$ is **[JEE 2008, 3 + 4 + 4 + 4]**

(A) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\, dt$ (B) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\, dt$

(C) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\, dt$ (D) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\, dt$

(b) Comprehension (3 questions together) :

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$

(i) If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ (B) $-\frac{4\sqrt{2}}{7^3 3^2}$ (C) $\frac{4\sqrt{2}}{7^3 3}$ (D) $-\frac{4\sqrt{2}}{7^3 3}$

(ii) The area of the region bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is

(A) $\int_a^b \frac{x}{3((f(x))^2 - 1)}\, dx + b f(b) - a f(a)$

(B) $-\int_a^b \frac{x}{3((f(x))^2-1)}dx + bf(b) - af(a)$

(C) $\int_a^b \frac{x}{3((f(x))^2-1)}dx - bf(b) + af(a)$

(D) $-\int_a^b \frac{x}{3((f(x))^2-1)}dx - bf(b) + af(a)$

(iii) $\int_{-1}^{1} g'(x)dx$ equals

(A) 2g(−1) (B) 0 (C) −2g(1) (D) 2 g(1)

10. Area of the region bounded by the curve y = ex and lines x = 0 and y = e is **[JEE 2009]**

(A) e − 1 (B) $\int_1^e$ In (e + 1 - y)dt

(C) e - $\int_e^1 e^x dx$ (D) $\int_1^e$ In ydy

11. Let the straight line x = b divide the area enclosed by $y = (1-x)^2$, y = 0, and x = 0 into two parts R_1 $(0 \le x \le b)$ and R_2 $(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals **[JEE 2011]**

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

12. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that f(x) = f(1 − x) for all $x \in [-1, 2]$.

Let $R_1 = \int_{-1}^{2} x\, f(x)\, dx$, and R_2 be the area of the region bounded by y = f(x), x = −1, x = 2, and the x-axis. Then **[JEE 2011]**

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$ (C) $2R_1 = R_2$ (D) $3R_1 = R_2$

13. Let S be the area of the region enclosed by $y = e^{-x^2}, y = 0, x = 0,$ and x = 1. Then **[JEE 2012]**

(A) $s \ge \frac{1}{e}$ (B) $s \ge 1 - \frac{1}{e}$

(C) $s \le \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$ (D) $s \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$